

# Vulnerable Markets

David Mayer-Foulkes, CIDE, Mexico

June 14, 2011

## Abstract

A production market with given preferences, technology and competition technology is *vulnerable* if it admits both perfect competition and monopoly or oligopoly. Under decreasing returns, the combination of sunk costs and a potential for monopoly profits can be sufficient basis for vulnerability, allowing a large agent to establish monopoly by installing enough productive capacity. The monopolist deters entry by threatening to oversupply the market. The threat is credible if the future discount rate is low enough and if reputation dynamics do not invite a slow loss of market power. Vulnerable markets allow financial institutions to concentrate ownership for profit.

## 1 Introduction

This paper asks the question: in an otherwise competitive context, can the size of the participants determine the structure of the market? More specifically, can a large participant take over a competitive market, impose higher prices and obtain a profit? The answer is clearly not if instantaneous production and sale are possible without sunk costs. Thus I study the case when production requires a sunk cost that serves to signal participants' intent to produce and makes them vulnerable to a loss. A large producer can then deter other participants by threatening to oversupply the market. When there is a potential for monopoly profits and the threat is credible, I show the large producer makes a profit over and above her expenditures on deterrence.

The paper demonstrates the existence of multiple types of market equilibria for a wide class of productive contexts with decreasing returns to scale, in which producers facing many buyers compete by selling their product. One type of equilibrium consists of many small owners. In the presence of a large enough agent, though, a monopoly equilibrium also exists. The equilibrium with many small owners is usually called perfect competition. However, the difference between the two types of equilibria resides in the distribution of *ownership* and not in the manner or technology of *competition*. Thus I call the equilibrium with many small owners *demopoly*<sup>1</sup>.

---

<sup>1</sup>Monopoly means "one seller," oligopoly "few sellers." Many sellers would be polipoly.

Under monopoly, an additional strategy appears for competing by selling a product than under demopoly. It is possible for a big producer to threaten other producers to oversupply the market so as to cause them a loss. We define the concept of *deterrent monopoly*, when an incumbent incurs a flow of expenditure to deter entry by other participants and therefore makes a profit. We define also the concept of *vulnerable market*, a market which has a demopolic (or competitive) equilibrium but that can also be captured by a monopoly or oligopoly. In the current paper, we show that the following four assumptions (in addition to decreasing returns) are sufficient for a market to be vulnerable to capture by a deterrent monopoly.

1) *Production involves sunk costs.*

An example is capital investment. As just mentioned, sunk costs make participants vulnerable to a threat.

2) *Aggregate sales' income rises when supply is decreased.*

This is a necessary condition for there to be an incentive to monopolize a market.

3) *The discount rate is such that the present value of future monopoly profits—net of deterrence costs—is larger than the sunk costs necessary to cover one round of competitive supply.*

This is necessary for the deterrence threat to be credible.

4) *Even a small deviation by a large agent from a costly credible threat will cause a discrete loss of reputation, that is, cause a discrete measure of small agents to disregard the threat in the future.*

This is a technical condition on how reputation works that does not provide incentives for an incumbent to slowly lose her monopoly over several production rounds rather than keeping to her stated threat of oversupplying the market.

Under these four conditions, demopoly will be the market equilibrium when all agents are small enough, and deterrent monopoly if there is a large enough agent. For simplicity, I leave the case of deterrent oligopoly for future work. Note that if an agent has enough credit, she will be big enough to establish a deterrent monopoly. It follows that for vulnerable markets such as this one, demopoly is incompatible with perfect credit.

The model I present is related to several strands of literature. One strand, with a long history, is the interaction between small and large players in a context of general equilibrium. Using the tools of cooperative game theory, Shitovitz (1973) follows Aumann (1964), as I do here, to model oligopolistic competition. The idea is to use a continuum of infinitesimal participants to represent small players (the *ocean*) and a finite set of participants with finite measures (*atoms*) to represent large players. Concentrating on the core of the economy, which is the set of allocations that no coalition can improve upon by using only their own resources, Shitovitz obtained counterintuitive results in which oligopolistic

---

Thus I use demopoly instead, meaning “the people sell,” on the suggestion of Sonia Di Gian-natale Menegalli.

outcomes are equivalent to competitive outcomes under quite wide assumptions. Okuno, Postlewaite and Roberts (1980) criticize this solution concept, obtaining quite different results for the Nash equilibrium in a noncooperative model of exchange. In their two commodity exchange economy, large agents or syndicated groups restrict supply to obtain higher welfare in a suboptimal equilibrium. Moreover, this approach also obviates the need to assume some agents are strategic while others are not. A related line of research, originated by Gabszewicz and Vial (1972), studies oligopoly ‘à la Cournot-Walras,’ with “few” oligopolists and “many” consumers. The original approach gives rise to a series of theoretical problems that have been successively overcome. Codognato and Gabszewicz (1991) define a Cournot-Walras equilibrium concept that does not depend on price normalization. Codognato (1995) and Codognato and Ghosal (2000) define instead a Cournot-Nash equilibrium concept that eliminates the asymmetry between strategic and non-strategic agents. Busetto, Codognato and Ghosal (2008) eliminate inconsistencies between these different types of models by harmonizing the number of stages and structure of play in the various settings. They then show that Cournot-Walras equilibria are equivalent to pseudo-Markov subgame perfect Nash equilibria, for which small players take only each commodity’s aggregate supply into account. Busetto, Codognato and Ghosal (2011) show the existence of a pure strategy Cournot-Nash equilibrium for a model of noncooperative exchange including large and small traders allowed to buy and sell all commodities.

The current paper also uses the subgame perfect Nash equilibria, but introduces threats in players’ strategies, not considered in the work we have mentioned. To do this it needs to introduce two additional elements, sunk costs and infinite repetition of the typical production round, so as to model the credibility of the threats. I therefore work with the simplest possible model. To be as realistic as possible, I keep to partial equilibrium in a single production market, rather than considering an exchange economy. As mentioned above, I also only consider the case of monopoly, leaving oligopoly for future work.

A second related strand of literature is predatory pricing, which in our model is the basis for the existence of deterrent monopoly. The predation literature has a long history. A series of works support the theoretical possibility of predation, while a series of critics doubt the practical relevance of these theories. McGee (1958) argues that purchase of rival firms is cheaper and more reliable than predatory pricing. Discussing predation and noting McGee (1958), Yamey (1972) concludes that predatory pricing should be given a place in the analysis of barriers to entry. Persson (2004) gives arguments in a multi-firm context as to why predation might be cheaper than mergers. The present day theory of predation is based on game theory, usually set in the context of imperfect competition (unlike our own context of small, competitive actors), and discusses the interaction of a series of market imperfections. For example Fudenberg & Tirole (1986) model a two-firm context in which predatory pricing has the function of jamming information for an entrant. Harrington (1989) proves that collusion and predation can exist in an oligopolistic setting. Bolton & Scharfstein (1990) show that predation can be motivated by financial contracting that threatens a

firm with bankruptcy if it fails to meet payments. Milgrom & Roberts (1997) show that predation can be rational for a monopolist deterring several potential entrants in a reputation setting. Roth (1996) shows that predatory pricing is rationalizable (a generalization of Nash equilibria based on modelling reasonable beliefs). It is interesting to note that predatory pricing is robustly reported in experimental economics (Gomez & Goeree, 2008).

Our theory of vulnerable markets in a sense falls within this tradition. The theoretical existence of predatory prices supports the theoretical existence of vulnerable markets. But there may be cheaper ways of deterring competition that are commonly available but involve additional market imperfections. Common examples could be local or targeted predatory pricing against specific entrants, small economies of scale in production or innovation, mergers, hostile takeovers, even physical threats and legal harassment. These would provide other, perhaps more realistic ways, in which vulnerable markets are captured by monopolies or oligopolies.

Indeed, the general structure of the argument for the existence of vulnerable markets is the following. A given good  $X$  is produced under conditions  $C$ . Suppose that under these conditions, as well as decreasing returns to scale, demopoly is possible and stable. All producers face the same risks and perils  $C$ . Nevertheless, it might still be possible for one or several large agents to decide to supply the full market for  $X$  monopolistically or oligopolistically, using the presence of conditions  $C$  to deter participation by small agents. Thus, conditions  $C$  are consistent with demopoly but can also serve to support monopoly or oligopoly. In the present case conditions  $C$  are the four conditions listed above, but other settings are conceivable.

While in this paper I examine only the case of monopoly, the literature on oligopolistic collusion supports the idea that the concept of vulnerable markets can be extended to the case of oligopoly. Abreu (1986), for example, shows that carrot and stick strategies can sustain oligopolistic collusion (optimally amongst symmetric strategies), which could be used to take over a vulnerable market. Collusion also exists under imperfect monitoring (Abreu, Pearce, and Stacchetti, 1986; Green and Porter, 1984). A series of other phenomena could also be involved, such as market share or price wars (Levenstein, 1993, Pot et al, 2010). Osterdal (2003) shows that the stability of carrot and stick collusion strategies (without the restriction to symmetry) improves with the degree of product differentiation. Thus a set of big players could collude to subdivide a set of product markets, allotting to each of themselves a subset of products on which to produce as a monopolist. It follows that in a general equilibrium setting a set of markets for which stable demopolic production is possible could be vulnerable to capture by monopolistic competition. Again, large financial institutions could provide the facilitating mechanisms for such a transformation from perfect to monopolistic competition, by providing the tools for a change in the ownership structure.

Finally, a discussion giving vulnerable markets an economic and historical setting is in order. First, market concentration has been the norm rather than the exception for US production during the 20th Century. From 1935 to 1992,

on average the four largest firms in 459 industries produced 38.4% of all shipments. Similarly, from 1992 to 2002, the 200 largest manufacturing companies accounted for 40% of manufacturing value added<sup>2</sup>. Consistently with market power, Hall (1988) shows in a study of US industry that marginal cost is often well below price. Industrial concentration has also been a salient feature of globalization. In 2007, 89.3% of global FDI inflows consisted of mergers and acquisitions (UNCTAD, 2008). By 2008, the world's top 100 non-financial transnational corporations produced 14.1% of global output (ibid). There is therefore a risk that global concentration could continue to rise towards US levels. Concentration has also risen tremendously in agriculture (Hendrickson & Heffernan 2007) and finance (D'Arista, 2009).

However, industrial concentration only began in the US near the end of the 19th Century. The Sherman Antitrust Act, meant to prevent the destruction of competition through the formation of cartels and monopolies, itself dates to 1890. This was the time when a wave of mergers radically transformed the banking sectors of Boston and Providence (Lamoreaux, 1991). In addition, the concentration of the banking system coincided with the first wave of mergers and acquisitions recognized by economic historians for the US, from 1893 to 1904.<sup>3</sup> This period saw the birth of the main steel, telephone, oil, mining, railroad and other giants of the basic manufacturing and transportation industries in the US. The major automobile manufacturers emerged during the second wave, from 1919 to 1929, featuring vertical integration that in the case of Ford reached all the way to the iron and coal mines. In the third period (1955 to 1969-73) the conglomerate concept took hold of American management. The fourth wave was the merger or takeover wave of the 1980s, which inaugurated the era of hostile takeover bids by major investment banks. It featured a new set of financial mechanisms such as junk bond financing and leveraged buyouts. The fifth wave (1993 to 2000) was the era of the mega-deal and took place under globalization. From a modest \$342 billion in 1992, worldwide merger volume reached \$3.3 trillion in 2000. This wave ended with the NASDAQ collapse, but by 2006 a sixth merger wave was in full swing. By mid-2008, just before the financial crisis, five banks were emitting 97 percent of all derivative assets.

Vulnerable markets are markets for which there is an incentive to change the ownership structure from demopoly to monopoly or oligopoly. The history of merger waves, which began when sufficiently large economic actors appeared, is consistent with the theory of vulnerable markets proposed here, as well as with the role that financial markets can play in transforming competitive into concentrated production ownership structures.

In what follows I present the deterrent monopoly model and conclude.

---

<sup>2</sup>Data from U.S. Census Bureau – Economic Census. 1992. “Concentration Ratios for the U.S.” <http://www.census.gov/epcd/www/concentration92-47.xls>.

<sup>3</sup>This paragraph's summary of merger waves is based on Lipton (2006).

## 2 The model

### 2.1 Production

In a partial equilibrium model, consider a single market for a given good  $X$ , to be sold at price  $P$ . Suppose that production of  $X$  occurs on a continuum of plants  $i \in [0, N]$ , where  $N$  will be determined endogenously. In each plant, production of a quantity  $q$  of good  $X$  costs  $c(q)$ , where  $c'' > 0$ , and for simplicity  $c' > 0$  for all quantities (alternatively  $c'$  could begin negative and end positive). Let the inverse demand for good  $X$  is given by

$$P = P(Q), P'(Q) < 0, \quad (1)$$

where  $Q = Nq$  is the total quantity produced.

The optimal arrangement for any level of aggregate production  $Q$  consists of operating each plant at its optimal capacity  $q^*$ , and choosing the number of plants  $N$  so that  $Q = Nq^*$ . This is because the first order condition for the minimization problem

$$\min_{s.t. Q=Nq} Nc(q),$$

after substituting for  $N$ , is

$$0 = \frac{d}{dq} \left[ \frac{Q}{q} c(q) \right] = \left[ -\frac{Q}{q^2} c(q) + \frac{Q}{q} c'(q) \right] \iff \frac{c(q)}{q} = c'(q). \quad (2)$$

Observe that since  $Q$  is fixed, the minimization problem is equivalent to minimizing average cost  $c(q)/q$ . The result is the well-known condition that average cost equals marginal cost, defining an optimal plant production level  $q^*$  where the elasticity of cost equals one,

$$\frac{q^* c'(q^*)}{c(q^*)} = 1.$$

The second order sufficient condition for the minimum is satisfied,

$$\left. \frac{d^2}{dq^2} \left[ \frac{Q}{q} c(q) \right] \right|_{q^*} = 2 \frac{Q}{q^3} c(q) - \frac{2Q}{q^2} c'(q) + \frac{Q}{q} c''(q) \Big|_{q^*} = \frac{Q}{q^*} c''(q^*) > 0.$$

Let the optimal average cost be

$$\gamma = \frac{c(q^*)}{q^*}.$$

### 2.2 Competitive production

Consider for simplicity the case of competitive production in which each plant is run by a distinct owner  $i \in [0, N]$ , so each firm corresponds to a single plant. Entry consists of establishing a new plant. Each firm maximizes profits

$$\pi_C(q) = Pq - c(q). \quad (3)$$

The first order condition for profit maximization of the price-taking firms is

$$0 = \pi'_C(q) = P - c'(q), \quad (4)$$

that is, marginal cost  $c'(q)$  equals marginal revenue  $P$ . Now firm entry reduces profits to zero defining a total number of firms  $N$  according to

$$0 = \pi_C = P(Nq)q - c(q). \quad (5)$$

Combining this with equation (2) yields

$$\frac{c(q)}{q} = P(Nq) = c'(q).$$

Hence  $q = q^*$ , which means competitive production is efficient, and the equilibrium number of firms  $N$  under perfect competition is:

$$N_C^* = \frac{1}{q^*} P^{-1}(\gamma).$$

### 2.3 Monopolistic production

Consider now a single owner who establishes a continuum of plants  $i \in [0, N]$  and prices the good as a monopoly. As noted above, minimizing production costs implies each plant operates at the optimum level of production  $q^*$ . Hence the monopolist chooses the total production quantity  $Q = Nq^*$  by choosing the number of plants  $N$  so as to maximize profits. Total cost can be written

$$C(Q) = Nc(q^*) = Q\gamma,$$

and marginal cost is  $C'(Q) = \gamma = \frac{c(q^*)}{q^*}$ . Total revenue can be written

$$TR(Q) = PQ = P(Q)Q.$$

Hence the monopolist maximizes

$$\max_Q \pi_M(Q) = TR(Q) - Q\gamma. \quad (6)$$

The first order condition occurs where marginal revenue equals marginal cost

$$TR'(Q) = \gamma = C'(Q). \quad (7)$$

Hence the quantity  $Q_M^*$  of production under monopoly is defined by

$$P(Q_M^*) + P'(Q_M^*)Q_M^* = \frac{c(q^*)}{q^*}.$$

Let us compare competitive and monopolistic production. The equations for quantities produced are:

$$P(Q_C^*) = \gamma \quad (8)$$

$$P(Q_M^*) + P'(Q_M^*)Q_M^* = \gamma \quad (9)$$

Since  $P'(Q_M^*) < 0$ , it follows  $P_C^* \equiv P(Q_C^*) < P(Q_M^*) \equiv P_M^*$  and hence  $Q_M^* < Q_C^*$ , monopolistic production is less than competitive production. For simplicity in what follows we now define units so that the optimal production at each plant at is  $q^* = 1$  unit of production.

## 2.4 Deterrent monopoly under sunk costs

Suppose now that establishing a production capacity requires input investments that will be lost even if production does not occur and that can therefore be lost in the process of competition. On this basis we define deterrent monopoly.

Specifically, suppose that each production round for  $X$  takes two periods. In the first period all producers, big and small, buy the inputs for the next period. In the next period they decide whether to dedicate their inputs fully to production or to sell the remaining inputs at a resale price which is a proportion  $0 < \kappa < 1$  of its purchase value, implying a loss representing the sunk cost.

A deterrent monopolist will be one that buys inputs for producing the full quantity  $Q_C^*$  of the competitive case. If further entry into production occurs, and the deterrent monopolist produces to capacity, the resulting price will be lower than the competitive price  $P_C^*$ . Every producer will loose. If for this reason there in fact is no additional entry, the deterrent monopolist need not produce the full amount  $Q_C^*$ , and can therefore obtain a profit. We examine the resulting game in the next section. For now we calculate the profit that occurs at this point if there is no entry. This results from maximizing

$$\max_Q \pi_D(Q) = TR(Q) - Q_C^* \gamma + \kappa(Q_C^* - Q) \gamma. \quad (10)$$

Writing  $Q_D^\kappa$  for the optimum production quantity for the deterrent monopolist, the first order condition yields

$$TR'(Q_D^\kappa) = \kappa \gamma. \quad (11)$$

We make the following assumptions on the total revenue function  $TR(Q)$ .

1)  $TR'(Q_C^*) < 0$ . This means that at the competitive equilibrium revenue can be increased by reducing the quantity sold.

And, for simplicity,

2) There is a unique  $\bar{Q}$  at which total revenue  $TR(\bar{Q})$  is maximal.

3) Marginal revenue is decreasing,  $TR''(Q) < 0$  on  $[0, Q_C^*]$ .

We can therefore obtain a unique solution for  $\bar{Q}$  satisfying  $0 < \bar{Q} < Q_C^*$ ,  $TR'(Q) > 0$  on  $[0, \bar{Q}]$ , and  $TR'(Q) < 0$  on  $[\bar{Q}, Q_C^*]$ .

The optimal production quantities  $Q_D^\kappa$  chosen by deterrent monopolists are decreasing in the resale rate  $\kappa$  and range monotonically over the interval  $[Q_M^*, \bar{Q}]$ , with  $Q_D^1 = Q_M^*$  and  $Q_D^0 = \bar{Q}$ . Let  $\pi_D^\kappa = \pi_D(Q_D^\kappa)$ . By the envelope theorem  $\frac{d}{d\kappa} \pi_D^\kappa = -(Q_C^* - Q_D^\kappa) \gamma < 0$ , profits rise as the optimal quantity  $Q_D^\kappa$  decreases, and this occurs as the resale rate  $\kappa$  increases. Prices  $P_D^\kappa$  move in the opposite direction. See Figure 1 for a price and quantity diagram under deterrent monopoly.

## 2.5 The deterrent monopoly game

We just saw that under the sunk cost conditions we defined, a large producer purchasing inputs for producing the full quantity  $Q_C^*$  can threaten other producers with losses if they enter production. This means that a game for market share can develop. We examine the case in which one big producer competes with a continuum of small producers.

The game consists of a repetition of production rounds. Each production round is the stage game of an infinitely repeated game. The players are Big, who is the large producer, and Small $_i$  a continuum of small players with  $i \in [0, N_{S \max}]$ , where the  $N_{S \max}$  is larger than  $N_C^*$ , the measure, or number of small players needed to supply the competitive market at optimal plant capacity<sup>4</sup>. Once we discuss the competitive case below and establish how the continuum of small players coordinates to participate, the game can be analyzed as a game with two players, Big and Small.

In the first period of each round, Big establishes a continuum of plants  $i \in [0, N_{1B}]$ , each with an optimal unit productive capacity. Then in period 2 she decides to operate  $N_{2B} \leq N_{1B}$  of these plants, at full capacity. Remaining inputs are sold at a loss proportional to  $1-\kappa$ . Meanwhile in the first period a continuum  $[0, N_{1S}]$  of small producers each establishes an infinitesimal plant with unit productive capacity.<sup>5</sup> In period 2 some set of small producers  $i \in [0, N_{2S}]$  decide to produce at unit capacity, while the remaining producers  $i \in [N_{2S}, N_{1S}]$  sell their inputs at a loss proportional to  $1-\kappa$ . All inputs will be used for production,  $N_{2S} = N_{1S}$ , when  $P \geq \kappa\gamma$  (even if  $P < \gamma$ ), because in this case if there is a loss from selling at less than market price it is still less than the loss incurred from reselling inputs. When  $P = \kappa\gamma$ , so both losses are equal, there will be some equilibrium production level  $0 < N_{2S} \leq N_{1S}$ , and when  $P \leq \kappa\gamma$ , reselling the inputs is preferable and  $N_{2S} = 0$ .

Introduce parameter  $t$  to indicate the round in the repeated game. The actions of players Big and Small $_i$  in the stage game are parametrized by  $(N_{1B}^t, N_{2B}^t)$ ,  $(N_{1S}^t, N_{2S}^t)$ . Quantities in the first period of each round  $t$  represent the quantities for which inputs were bought, while quantities in the second period represent the actual quantities produced.

### 2.5.1 The stage game

The main asymmetry between small players and Big is size, and the implication that size has on strategic play. Small players cannot affect prices, because the quantities they produce are too minute, and therefore do not affect Big. Only the non-cooperative collective decisions taken by all small players together affect Big. Even so, it is possible for small players to have strategies, as we shall see below.

---

<sup>4</sup>Abusing language, I will use *number* instead of *measure* for small producers. Thus  $N_S$  small producers refers to a set of measure  $N_S$  of small producers, usually  $[0, N_S]$ .

<sup>5</sup>For simplicity we assume that the small, infinitesimal producers are of equal size. It would not be difficult to introduce a measure representing different sizes.

The stage game of our infinitely repeated game consists of the two period rounds that we have described, written as if they occurred in one period at time  $t$ . We write the game as if there were two players, Big and Small, where the actions taken by Small are the result of the actions of all small players. The actions available to each player are

$$\{(N_{1B}^t, N_{2B}^t) : 0 \leq N_{2B}^t \leq N_{1B}^t \leq N_C^*\}, \quad (12)$$

$$\{(N_{1S}^t, N_{2S}^t) : 0 \leq N_{2S}^t \leq N_{1S}^t \leq N_{S \max}\}. \quad (13)$$

We set  $N_C^*$  as the upper limit for  $N_{1B}^t$ , and also usually for  $N_{1S}^t$ , because there is no incentive to set up productive capacities above levels for which the profit is certain to be zero (something we verify in the exposition below when relevant). The stage game payoffs are as follows. Big's payoff  $\pi_B^t$  is

$$\pi_B^t = P(N_{2B}^t + N_{2S}^t)N_{2B}^t - \gamma N_{1B}^t + \kappa\gamma(N_{1B}^t - N_{2B}^t).$$

where  $P^t = P(N_{2B}^t + N_{2S}^t)$  is the price of good  $X$ , which depends on Big's own level of production and on aggregate small producers' production  $N_{2S}^t$ . The discount rate between periods 1 and 2 is assumed to be negligible. The payoff obtained by small player  $i \in [0, N_{1S}^t]$  in period 2 depends on whether she produces or not,

$$\pi_S^t(i) = \begin{cases} P(N_{2B}^t + N_{2S}^t) - \gamma, & i \in [0, N_{2S}^t], \\ -(1 - \kappa)\gamma, & i \in (N_{2S}^t, N_{1S}^t]. \end{cases}$$

If some small players produce and others do not then these two quantities will be equal.

### 2.5.2 Small players with competitive market strategy

One of the properties of perfect markets is that they generate coordination (the *invisible hand*). Each producer observes prices and decides how much to supply according to her production possibilities, and an efficient equilibrium is generated. Once producers know the price with certainty, knowledge about the strategies of others, or past "play of the game" is irrelevant. We begin by observing how perfect competition works out in our model when only small players participate, and then sequentially construct new strategic situations that develop when Big participates.

Because we assume identical small producers, a coordination problem arises in deciding which producers participate and which do not. When for example we say that  $N_C^*$  is the number of small players that clears the competitive market, and mean that small producers  $i \in [0, N_C^*]$  each produce one unit, while potential participants  $i \in [N_C^*, N_{S \max}]$  do not participate in production, implicitly we are assuming that small players take their decisions instantaneously but in increasing order along the interval  $[0, 1]$ . We make this assumption for how decisions are taken by the small agents. For example, it could be that as  $i$  increases, producers are negligibly less productive.

### 2.5.3 Perfect markets

Consider the case when there are only small producers, so  $N_{1B}^t = N_{2B}^t = 0$ . The  $i^{th}$  player therefore has the strategy “purchase inputs to produce a unit of  $X$  if the expected price  $P(i)$  is higher than the production cost  $\gamma$ , otherwise do not purchase inputs.” Recall that when this producer decides to participate the whole interval  $[0, i]$  of producers is participating. Since the inverse demand function  $P(i)$  is decreasing in  $i$ , the measure of small participants purchasing inputs in period 1 to produce one unit of  $X$  in period 2 is  $N_{1S}^t = N_C^*$ . When period 2 arrives, all of the small producers know that the price  $P$  cannot fall below  $\gamma$ , so none decides to curtail production and  $N_{2S}^t = N_{1S}^t = N_C^*$ . Hence

$$\pi_S^t(i) = P(N_{1S}^t) - \gamma = 0, \quad i \in [0, N_{1S}^t]. \quad (14)$$

This is a Nash equilibrium. As we saw, none of the producers has an incentive to change their decision in period 2, so Small has no incentive to change  $N_{2S}^t$ . It follows none of the producers  $i$  has an incentive to change their decision in period 1. If  $i < N_{1S}^t$ ,  $\pi_S^t(i) = 0$  (actually we are assuming this is negligibly positive), while if  $i > N_{1S}^t$ , since by assumption  $i$  only participates if all producers in interval  $[0, i]$  also participate, making  $\pi_S^t(i) = P(i) - \gamma < 0$ . Note that we are only interested in asking whether there is a deviation  $N_{1S}^{t'}$  from  $N_{1S}^t$ , so we could properly argue that if  $N_{1S}^{t'} < N_{1S}^t$  then deviating producer  $i = N_{1S}^{t'}$  has only ceased to participate if producers  $(N_{1S}^{t'}, N_{1S}^t]$  are not producing, in which case profits would be  $\pi_S^t(i) = 1 - N_{1S}^{t'} - \gamma > 0$ , which deviating player  $i = N_{1S}^{t'}$  would be forsaking to earn 0 instead.

### 2.5.4 Big comes in

Now consider the case when small producers keep to the competitive market strategy but a large player participates in the market. Suppose the large player, Big, sets up in period 1 a productive capacity sufficient to supply the full competitive market but then in period 2 underproduces to raise the price. If small producers keep to their original competitive strategy, they will not purchase any inputs and Big’s profits are unimpeded.

However, there is a new element to strategy here. Big is no longer acting exclusively according to expected price. She is elbowing out other participants. The way this occurs is that her intended productive capacity, as measured by purchased inputs, remains fixed independently of the intentions of small agents. We can model this by supposing that Big announces her productive capacity before the small agents, or by supposing that all players have a last ditch opportunity to cancel their input purchases after everybody announces their intentions, Big refusing to cancel and thus small players cancelling instead. Below we model this properly as equilibria in a game. The point however is to note that by including knowledge of other player’s strategies we are actually starting to go beyond the simple, innocent competitive market strategy.

In this protogame, for any announcement  $N_{1B} \leq P^{-1}(\gamma) = N_C^*$  made by Big,  $N_{1S} = N_C^* - N_{1B}$  small producers purchase inputs to produce a unit of

good  $X$  in period 2. Then if Big increases the price by reducing production, small agents produce to full capacity and Big maximizes

$$\pi_B(N_{1B}, N_{1S}, N_{2B}) = P(N_{1S} + N_{2B})N_{2B} - \gamma N_{1B} + \kappa\gamma(N_{1B} - N_{2B}).$$

Define the total revenue function

$$TR(N_{1S}, N_{2B}) = P(N_{1S} + N_{2B})N_{2B}. \quad (15)$$

For simplicity we assume this function is concave in  $N_{2B}$  for any  $N_{1S}$ . That is,

$$\frac{\partial^2 TR}{\partial N_{2B}^2}(N_{1S}, N_{2B}) = P''(N_{1S} + N_{2B})N_{2B} + 2P'(N_{1S} + N_{2B}) < 0.$$

We can now determine what the optimal production levels for Big are for different levels of small participants.

**Lemma 1** *Optimal Production Levels.* Suppose that inputs purchased for production by Big and Small do not on their own oversupply the market, but jointly cover or exceed the competitive level of supply, that is,  $N_{1B} \leq N_C^*$ ,  $N_{1S} \leq N_C^*$  and  $N_{1B} + N_{1S} \geq N_C^*$ . There is some maximal number of small participants  $\bar{N}_{1S} \in (0, N_C^*)$  for which when  $N_{1S} < \bar{N}_{1S}$  Big's optimal production level is to undersupply the market by choosing  $N_{2B}^* \in (0, N_C^* - N_{1S})$ . In these cases, if  $N_{1B} + N_{1S} = N_C^*$ , small players will dedicate all of their inputs to production,  $N_{2S} = N_{1S}$ , and obtain a proportionally higher profit than Big, free-riding on her underproduction. If instead  $N_{1S} > \bar{N}_{1S}$ , Big's optimal response always yields a negative payoff unless  $N_{1B} + N_{1S} = N_C^*$ , in which case all inputs are assigned to production, so  $N_{2B} = N_{1B}$ , and the payoff is zero for both Big and Small. When input purchases imply oversupply, so  $N_{1B} + N_{1S} > N_C^*$ , payoffs remain invariant if all input resales are carried out by Big, so  $N_{2S} = N_{1S}$  can be assumed. In the case  $N_{1S} = \bar{N}_{1S}$ , the optimal response  $N_{2B} = N_C^* - N_{1S}$  yields competitive market prices. Big's payoffs are decreasing in Small's participation and in her own initial input purchase.

**Proof.** Big's optimization problem is

$$\pi_B^*(N_{1B}, N_{1S}) = \max_{N_{2B} \leq N_{1B}} P(N_{1S} + N_{2B})N_{2B} - \gamma N_{2B} - (1 - \kappa)\gamma(N_{1B} - N_{2B}). \quad (16)$$

Observe that

$$\left. \frac{\partial \pi_B}{\partial N_{2B}} \right|_{N_{2B}=0} = P(N_{1S}) - \kappa\gamma > P(N_C^*) - \kappa\gamma = (1 - \kappa)\gamma > 0.$$

On the other hand, at  $N_{1S} + N_{2B} = N_C^*$ , Big's marginal profit is:

$$\left. \frac{\partial \pi_B}{\partial N_{2B}} \right|_{N_{2B}=N_C^*-N_{1S}} = P'(N_C^*)N_{2B} + \gamma - \gamma + (1 - \kappa)\gamma \quad (17)$$

$$= P'(N_C^*)(N_C^* - N_{1S}) + (1 - \kappa)\gamma. \quad (18)$$

$$\begin{cases} < -P(N_C^*) + (1 - \kappa)\gamma = -\kappa\gamma < 0 & \text{for } N_{1S} = 0 \\ = (1 - \kappa)\gamma > 0 & \text{for } N_{1S} = N_C^* \end{cases} \quad (19)$$

In addition, the expression is monotonic, since

$$\frac{d}{dN_{1S}} \left( \frac{\partial \pi_B}{\partial N_{2B}} \Big|_{N_{2B}=N_C^*-N_{1S}} \right) = -P'(N_C^*) > 0. \quad (20)$$

Hence there is a unique value  $\bar{N}_{1S} \in (0, N_C^*)$  below which the maximum payoff for Big is interior and above which no undersupply is worthwhile. Since setting  $N_{2B} = N_{1B}$  yields a zero payoff, when the maximum is interior, i.e. when  $N_{1S} < \bar{N}_{1S}$ , profits  $\pi_B^*(N_{1B}, N_{1S})$  are positive. Consider now the case when  $N_{1S} + N_{2B} > N_C^*$ . Concavity of the total revenue function implies that if  $N_{1S} \leq \bar{N}_{1S}$ , the optimal  $N_{2B}$  remains unchanged. If instead  $N_{1S} > \bar{N}_{1S}$ , Big's payoffs increase with oversupply, implying the optimal value is negative. In these cases if selling the product is still better than reselling the inputs, so  $P(N_{1S} + N_{2B}) > \kappa\gamma$ , small producers set  $N_{2S} = N_{1S}$ . Otherwise Big limits overproduction so  $P(N_{1S} + N_{2B}) = \kappa\gamma$ . Big can always do this, because  $N_{1S} \leq N_C^*$ . Since at this price selling the product or reselling the inputs yields the same payoffs, we can assume  $N_{2S} = N_{1S}$ . Note also by the envelope theorem that

$$\frac{\partial \pi_B^*}{\partial N_{1S}}(N_{1B}, N_{1S}) = P'(N_{1S} + N_{2B}) N_{2B} < 0 \quad (21)$$

$$\frac{\partial \pi_B^*}{\partial N_{1B}}(N_{1B}, N_{1S}) = -(1 - \kappa)\gamma < 0. \quad (22)$$

■

Note that if the number of small participants is less than  $\bar{N}_{1S}$ , Big can still make a profit by undersupplying the market. Write

$$N_{DF}^*(N_{1S}) = N_{2B}^*(N_{1S}) + N_{1S} < N_C^*, \quad (23)$$

$$P_{DF}^*(N_{1S}) = P(N_{DF}^*(N_{1S})) > \gamma \text{ if } N_{1S} < \bar{N}_{1S} \quad (24)$$

$$\pi_{BDF}^*(N_{1S}) = P(N_{DF}^*) N_{2B}^* - \gamma N_{2B}^* - (1 - \kappa)\gamma(N_{1B} - N_{2B}^*) \quad (25)$$

for the total quantity supplied to the market in this case, the resulting price, and Big's profit when  $N_{1B} = N_C^* - N_{1S}$ . The dependence on  $N_{1S}$  will be omitted from the notation unless required, as in the last line. "DF" means "deterrent with free riders." Big's profit is positive if the input purchase does not already oversupply the market, that is,  $\pi_{BDF}^*(N_{1S}) > 0$  if  $N_{1B} + N_{1S} = N_C^*$ . In this case the small participants make a proportionally higher profit, because they do not have to resell inputs at a loss.

In the game below, Big will dissuade small participation by threatening to oversupply the market and therefore causing everybody a loss. What is Big's optimal choice for production in this case? Suppose  $N_{1S} + N_{1B} \geq N_C^*$ . Then Big's optimization problem for punishing is:

$$\begin{aligned} & \max_{N_{2B} \leq N_{1B}, P(N_{1S} + N_{2B}) \leq \gamma} \pi_B^{\text{punish}} = \\ & = P(N_{1S} + N_{2B}) N_{2B} - \gamma N_{2B} - (1 - \kappa)\gamma(N_{1B} - N_{2B}). \end{aligned} \quad (26)$$

The objective function is the same profit function as in Lemma 1. The only change is the addition of the restriction  $P(N_{1S} + N_{2B}) \leq \gamma$ .

**Corollary 2** *Cheapest Punishment.* Suppose that inputs purchased for production by Big and Small do not on their own oversupply the market, but can jointly oversupply the market, that is,  $N_{1B} \leq N_C^*$ ,  $N_{1S} \leq N_C^*$  and  $N_{1B} + N_{1S} > N_C^*$ . When  $N_{1S} \leq \bar{N}_{1S}$ , Big's highest payoff inflicting punishment occurs with slight oversupply, that is  $N_{2B} = (N_C^* - N_{1S})^-$ . If instead  $N_{1S} > \bar{N}_{1S}$ , Big's optimal response already produces a negative payoff for both players. Note therefore that

$$\begin{aligned} & \pi_{BDF}^* - \pi_B^{Punish} = \\ = & \begin{cases} (P(N_{DF}^*) - \gamma)N_{DF}^* - (1 - \kappa)\gamma(N_{DF}^* - N_C^*) > 0 & N_{1S} < \bar{N}_{1S} \\ 0 & N_{1S} \geq \bar{N}_{1S} \end{cases} \end{aligned} \quad (27)$$

The immediate gain  $\pi^{DEV}$  that Big obtains from deviating from punishment,

$$\pi^{DEV}(N_{1S}) = \pi_{BDF}^*(N_{1S}) - \pi_B^{Punish}(N_{1S}) \quad (28)$$

is independent of  $N_{1B}$ , and decreasing in  $N_{1S}$ .

**Proof.** The result follows from the signs for  $\left. \frac{\partial \pi_B}{\partial N_{2B}} \right|_{N_{2B}=N_C^*-N_{1S}}$  established in the proof of the previous lemma. Note for reference that

$$\begin{aligned} & \pi_{BDF}^* - \pi_B^{Punish} = \\ = & P(N_{DF}^*)(N_{DF}^* - N_{1S}) - \gamma(N_{DF}^* - N_{1S}) - (1 - \kappa)\gamma(N_{1B} - (N_{DF}^* - N_{1S})) \\ & - \{P(N_C^*)(N_C^* - N_{1S}) - \gamma(N_C^* - N_{1S}) - (1 - \kappa)\gamma(N_{1B} - (N_C^* - N_{1S}))\} \\ = & (P(N_{DF}^*) - \gamma)(N_{DF}^* - N_{1S}) - (1 - \kappa)\gamma(N_C^* - N_{DF}^*) > 0. \end{aligned} \quad (29)$$

because  $\pi_B^{Punish} < \pi_B^{Profit}$  by construction,  $\gamma < P(N_{DF}^*)$ ,  $N_{DF}^* - N_{1S} = N_{2S}^* > 0$  and  $N_{DF}^* < N_C^*$ . ■

### 2.5.5 Small gets wise

What happens when small producers observe Big can increase prices in period 2 and make a profit? Small producers must now decide to abandon the simple rules of participation in perfectly competitive markets, to become strategic players. Now both types of players observe each other's strategies. Small producers have some incentives to participate in production, because even though Big purchases inputs to supply the full competitive market, if she then undersupplies the market, small participants will make a profit. Therefore, Big must credibly threaten small participants to oversupply the market if they participate, lowering price  $P$  below the competitive market level so as to impose a loss for participating. To show Big's threat is credible, we turn to the infinitely repeated game with perfect monitoring and show the existence of subgame perfect Nash equilibria.

Now, we nevertheless assume that anytime Big does not purchase enough inputs to supply the full competitive market in the first period, small agents will be aware of this and purchase inputs to complete this supply, with the effect that they will make at least the normal profit (zero) in any circumstance. We thus assume that after Big has purchased inputs, small agents have a last chance to purchase inputs as well.

We thus consider two types of small players, *tough* small players with a strategy, and an additional set of price taking, *competitive* small producers.

Big's strategy for making a profit depends on her reputation for keeping to her punishment strategy, that is, on small players' belief that Big will keep to her punishment strategy. Suppose the strategies of Big and Small are the following. Big will in each production round purchase inputs to produce at the competitive level. If small producers participate, Big will keep the price of good  $X$  below the competitive level, while if they do not, she will reduce production to the deterrent monopoly level, making a profit. In turn, small producers decide not to participate in production. However, I assume that if ever Big deviates from her announced punishment strategy, even by an arbitrarily small amount, this produces a wave of new entrants (that is not arbitrarily small). Specifically, if after a measure  $N_{1S}$  of small producers purchases inputs, Big still undersupplies the market so that  $N_{2B} + N_{1S} < N_C^*$ , I assume that her reputation for toleration of small producers (the opposite of toughness) increases further by a discrete amount  $\eta > 0$ , and from then on a larger set of small producers  $[0, N_{1S} + \eta]$  become tough and believe that they can participate without punishment. In addition, if Big does not purchase enough inputs in period 1 to produce at the competitive level, then enough additional small competitive producers purchase inputs to complete the competitive level of aggregate production, further raising the perceived level of Big's tolerance. That is, for any given realization of the game, the latest level of reputation for Big's tolerance  $T^t$  for participation of small players is given by:

$$T^1 = 0, T^{t+1} = \begin{cases} T^t & \text{if } N_{1S}^t \leq T^\tau, \\ N_{1S}^t + \eta & \text{if } N_{1S}^t > T^\tau, \end{cases} \quad t = 1, 2, \dots \quad (30)$$

For any time  $t$  the game is characterized by the initial established toleration level  $T^t$ . To examine whether the strategies form a subgame perfect equilibrium we in effect have to examine all initial levels  $T^t$ . For Big, it is only worth punishing if the profits that can be obtained from compliance are high enough. This means that the future discount rate  $\delta < 1$  must be high enough (depreciation low enough) for the total expected flow of profits to be high enough, an application of the Folk Theorem for infinitely repeated games. I use Ray's (2003) summary of the one-shot deviation principle as reference for proving the strategies form a subgame perfect equilibrium.

What I will show is that for high enough  $\delta$ , an initial toleration levels  $T^1 = 0$  yields a Nash equilibrium, but that there is a maximum toleration level  $T_\delta \in (0, \bar{N}_{1S})$  above which profits become too low to warrant a credible threat. Recall  $\bar{N}_{1S}$  is the number of tough small free riders at which Big's profits reduce to

zero. Thus if  $T^t$  ever rises above  $T_\delta$ , there will be at least  $\tilde{N}_{1S}$  small participants, because at lower levels Big is sure to undersupply, so that all plants will make a profit, and punishment is no longer a credible threat. For  $T^t > T_\delta$  there will be a competitive Nash equilibrium for any pair of numbers

$$(\tilde{N}_{1B}, \tilde{N}_{1S}) : \tilde{N}_{1S} \geq \tilde{N}_{1S}, \tilde{N}_{1B} = N_C^* - \tilde{N}_{1S} \geq 0 \quad (31)$$

representing a number of Small and Big's participation.

We can now summarize the above mentioned strategies as follows. For  $T^t \leq T_\delta$ ,

$$\begin{aligned} N_{1B}^t &= N_C^* - T^t, \\ N_{1S}^t &= \max\{N_C^* - N_{1B}^t, T^t\}, \\ N_{2B}^t &= \begin{cases} N_{2B}^*(N_{1S}^t) \text{ (undersupply)} & \text{if } N_{1S}^t \leq T^t \\ N_C^* - N_{1S}^t \text{ (punishment)} & \text{if } N_{1S}^t > T^t \end{cases} \\ N_{2S}^t &= N_{1S}^t. \end{aligned} \quad (32)$$

Note that strategy in period 1 is written with Small having a last round of decision after Big in case Big underinvests. Similarly, period 2 is written as a function of any arbitrary play in period 1 satisfying  $N_{1S}^t + N_{1B}^t \geq N_C^*$  (because otherwise even competitive small players are losing an opportunity to participate), so as to be able to begin the game at any period of the stage game, as required in the evaluation of one-shot deviations. We saw in Lemma 1 and its Corollary that whether Big punishes or undersupplies to make a profit, Small can be considered to choose  $N_{2S}^t = N_{1S}^t$ . Finally, for  $T^t > T_\delta$ , choosing any pair of numbers  $(\tilde{N}_{1B}, \tilde{N}_{1S})$  as described above (31),

$$\begin{aligned} N_{1B}^t &= \tilde{N}_{1B}, \\ N_{1S}^t &= \tilde{N}_{1S}, \\ N_{2B}^t &= N_{1B}^t, \\ N_{2S}^t &= N_{1S}^t. \end{aligned} \quad (33)$$

Note that the game is written as a two player game. Player Small reflects the aggregate behavior of the continuum of small players  $\text{Small}_i$   $i \in [0, N_{S \max}]$ , according to the coordination rules described from perfect competition.

**Theorem 3** *Strategies (32), (33) define a subgame perfect Nash equilibrium. While demopoly is possible, a large enough agent can establish deterrent monopoly.*

The proof is in the subsections that follow. We first write down the payoffs, then examine possible deviations within the stage game, and finally show that there are no profitable one-shot deviations.

### 2.5.6 Payoffs

The payoffs for stage  $t$  of the game given an initial tolerance level  $T^t$  are the following. So long as  $T^t < T_\delta$ , Big's payoff is  $\pi_{BDF}^*(T^t)$ , see (25), and each  $\text{Small}_i$ 's payoff is  $P(N_{DF}^*) - \gamma$ , see (24) and recall  $q^* = 1$ . If instead  $T^t \geq T_\delta$ , all payoffs are 0. It follows that given a discount factor  $\delta < 1$  the payoff of the infinitely repeated game is  $(1 - \delta)^{-1}$  times these payoffs.

### 2.5.7 Deviation payoffs

We now examine stage  $t$  of the game for differences in payoffs that might be relevant for our examination of one-shot deviations below. The only deviation that affects future play is when Big does not punish. Thus we need only examine two kinds of deviations, those that raise payoffs at stage  $t$ , and those when Big reneges from punishing.

First we examine deviations by Big. Suppose  $T^t < T_\delta$ . Big can deviate in the first period by arbitrarily setting some  $N_{1B}^t$ . If Big attempts to undersupply the market setting  $N_{1B}^t < N_C^* - T^t$ , then Small reacts by completing its supply with  $N_{1S}^t = N_C^* - N_{1B}^t > T^t$ . In this case Big suffers a double loss. First, her optimal stage  $t$  payoff is now lower, because  $\pi_{BDF}^*(N_{1S}^t)$  is a decreasing function of  $N_{1S}^t$ . Second, she cannot punish Small, because she cannot oversupply the market. Thus her reputation suffers, shifting to  $T^{t+1} = T^t + \eta$ , which implies future payoffs are lower.

If instead Big oversupplies tough small producers with  $N_{1B}^t > N_C^* - T^t$  (even in the case  $T^t = 0$ ), these players will still put all their inputs to production, so  $N_{2S}^t = N_{1S}^t = T_{1S}^t$ , and since Big's payoff will be worse than at  $N_{1B}^t = N_C^* - T_{1S}^t$ , since it is decreasing in  $N_{1B}^t$  because more inputs will have to be sold at a loss.

Alternatively, Big deviates in the second period, after any given play  $(N_{1B}^t, N_{1S}^t)$ , which now serves as initial point of the game for one shot deviations. Recall we need only consider input purchases satisfying  $N_{1S}^t \geq N_C^* - N_{1B}^t$  and Small production decisions  $N_{2S}^t = N_{1S}^t$ .

In considering deviations, since we have already chosen optimal play following Lemma 1 and its Corollary, there are only two deviations left to consider, deciding to punish when no punishment is called for, for which there are no incentives since this is costly and has no implications for future play, or deciding not to punish when punishment is called for. We saw that this will only present a gain for Big when  $N_{1S} < \bar{N}_{1S}$ , given by (29).

Suppose now  $T^t \geq T_\delta$ . Big's tolerance is now so high that punishment is meaningless. Big would only make a profit if Small played  $N_{1S} < \bar{N}_{1S}$ , but then so would Small, so there will be at least  $\bar{N}_{1S}$  small participants. Thus Big can in no circumstance expect a positive profit, so cannot do better than participating with a  $0^+$  payoff. Her only alternative is to reduce her participation, something which offers no gain.

Now let us examine deviations by Small. Suppose  $T^t \leq T_\delta$  and Small plays  $N_{1S}^t$ . If some of the tough players are not participating, so  $N_{1S}^t < T^t$ , profits increase for Big and for the remaining small participants, but decrease for those small players who have forgone participation, with no gain in the future. If instead  $N_{1S}^t > T^t$ , then Big punishes and all small players experiences a loss, again with no gain in the future. Therefore none of these cases need be considered in one shot deviations for the present game. There are also no profitable deviations for Small in the second period or in the case  $T^t > T_\delta$ .<sup>6</sup>

---

<sup>6</sup>We do not discuss additional strategies Big could have to throw off tough small players, such as targetted price wars.

### 2.5.8 One shot deviations

Our analysis of deviations in the stage game shows that only one profitable candidate for a one shot deviation exists, apart from the possibility of shifting  $T_\delta$ , which we examine below. This is for Big not to punish in the second round. To see if this is profitable, the immediate gain must be compared with the loss in future play. Suppose therefore  $T^t \leq T_\delta$ , and small plays  $N_{1S}^t > T^t$ . We saw that Big has one optimal choice for punishing, which consists of just oversupplying the market. The value  $V$  of deviating by not punishing is:

$$V = \begin{cases} \pi^{DEV}(N_{1S}^t) + \frac{\delta}{1-\delta} (\pi_{BDF}^*(N_{1S}^t + \eta) - \pi_{BDF}^*(T^t)) & \text{if } N_{1S}^t + \eta \leq T_\delta, \\ \pi^{DEV}(N_{1S}^t) - \frac{\delta}{1-\delta} \pi_{BDF}^*(T^t) & \text{if } N_{1S}^t + \eta > T_\delta. \end{cases} \quad (34)$$

For  $V \leq 0$  (we consider there is no deviation if the gain is zero) what is needed is:

$$\frac{\delta}{1-\delta} \geq \max \left\{ \begin{array}{l} \sup_{T^t \leq N_{1S}^t \leq T_\delta - \eta} \frac{\pi^{DEV}(N_{1S}^t)}{\pi_{BDF}^*(T^t) - \pi_{BDF}^*(N_{1S}^t + \eta)}, \\ \sup_{N_{1S}^t + \eta > T_\delta} \frac{\pi^{DEV}(N_{1S}^t)}{\pi_{BDF}^*(T^t)} \end{array} \right\} \quad (35)$$

$$= \max \left\{ \frac{\pi^{DEV}(0)}{\pi_{BDF}^*(0) - \pi_{BDF}^*(\eta)}, \frac{\pi^{DEV}(0)}{\pi_{BDF}^*(T_\delta)} \right\}. \quad (36)$$

We are using here the results that  $\pi^{DEV}$  and  $\pi_{BDF}^*$  are decreasing functions of  $N_{1S}^t$  (see Lemma 1 and its Corollary). The subgame perfect Nash equilibrium is possible for any  $\delta$  satisfying  $\frac{\delta}{1-\delta} \geq \frac{\pi^{DEV}(0)}{\pi_{BDF}^*(0) - \pi_{BDF}^*(\eta)}$ , by choosing  $T_\delta$  large enough that  $\frac{\delta}{1-\delta} = \frac{\pi^{DEV}(0)}{\pi_{BDF}^*(T_\delta)}$ . At this level of  $T_\delta$  Big has no incentive to deviate from  $T_\delta$ , because she would forfeit credible threats that would yield profits for games starting with relatively low reputations. Note the crucial role of  $\eta > 0$  for the existence of  $\delta < 1$  satisfying the first inequality, meaning that Big suffers a finite rather than infinitesimal loss of reputation even if Small oversteps the limit  $T^t$  by an arbitrarily small amount.

## 2.6 Two Corollaries

We can write down the following corollary on how financial development can contribute to diminishing competition.

**Corollary 4** *Suppose aggregate financial development is high enough that financial institutions can borrow resources  $N_C^*$  from small players and lend them to an agent Big at a total transaction cost  $TC$  less than the deterrent rate of profit  $\pi_{BDF}^*(0)$ . Then Big establishes a deterrent monopoly.*

**Proof.** Small players will prefer to lend financial institutions the resources  $N_{1S}^t = N_C^*$  that they would dedicate to production so long as the aggregate payoff  $\pi_S$  offered to them is positive. Big will establish the deterrent monopoly

so long as the total interest  $R$  she has to pay on these resources satisfies  $0 < R < \pi_{BDF}^*(0)$ . Financial institutions will take care of these transactions so long as  $R > \pi_S + TC$ . Since  $TC < \pi_{BDF}^*(0)$  amounts  $\pi_S$  and  $R$  exist satisfying these restrictions. ■

The deterrent monopoly payoff minus transaction costs,  $\pi_{BDF}^*(0) - TC$  will be distributed between Big, the financial system and small agents according to the prevailing financial market structure and the agents' bargaining power. As compared to demopoly, the resulting monopolic productive system will be inefficient because it will 1) supply less than the optimal amount of good X, 2) at a higher than optimal price, and 3) it will waste resources on deterrence during every production round. In addition, as compared to demopoly, monopoly will transfer wealth to the financial system and to Big.

We can also note the following strategy for preventing monopolization of good X that is available to a government that can produce competitively.

**Corollary 5** *The public option. Suppose the government produces in  $T_\delta$  plants, selling at a competitive price. Then Big will not establish a deterrent monopoly.*

**Proof.** Big has no incentives to establish a deterrent monopoly if her maximum market share is less than  $N_C^* - T_\delta$ . ■

Note that once the government establishes the public option, or subcontracts it to small agents, it is the market that establishes the competitive price.

### 3 Conclusion

We have shown the theoretical existence of vulnerable markets for a wide class of production contexts. These are markets that can support both perfect market equilibria or equilibria with market power, in our case deterrent monopoly. In these markets, the only a priori characteristic that distinguishes the two equilibria is the ownership structure. The technologies for production and competition, as well as consumer preferences remain unchanged. Big agents, of course, have recourse to strategies that are unavailable to small agents and that depend on impacting the whole market.

We have also shown that the existence of vulnerable markets provides incentives for financial institutions to concentrate production. When this occurs, the resulting monopolic equilibrium is less efficient, wastes resources on deterrence, and transfers resources to financial institutions and large agents.

## 4 References

Abreu, Dilip (1986). "Extremal equilibria of oligopolistic supergames," *Journal of Economic Theory*, Elsevier, vol. 39(1), pages 191-225, June.

Abreu, D., Pearce, D. and Stacchetti, E. (1986): "Optimal Cartel Equilibria with Imperfect Monitoring," *Journal of Economic Theory*, 39, 251-269.

Aumann, R.J. (1964). "Markets with a continuum of traders," *Econometrica* 32,39-50.

Busetto, Francesca & Codognato, Giulio & Ghosal, Sayantan (2008). "Cournot-Walras Equilibrium as a Subgame Perfect Equilibrium," *The Warwick Economics Research Paper Series (TWERPS) 837*, University of Warwick, Department of Economics.

Bolton, Patrick & Scharfstein, David S. (1990). "A Theory of Predation Based on Agency Problems in Financial Contracting," *The American Economic Review*, Vol. 80, No. 1 (Mar., 1990), pp. 93-106.

Busetto, Francesca & Codognato, Giulio & Ghosal, Sayantan (2011). "Non-cooperative oligopoly in markets with a continuum of traders," *Games and Economic Behavior*, Elsevier, vol. 72(1), pages 38-45, May.

Codognato G. (1995). "Cournot-Walras and Cournot equilibria in mixed markets: a comparison," *Economic Theory* 5, 361-370.

Codognato G, Gabszewicz JJ (1991). "Equilibres de Cournot-Walras dans une économie d'échange," *Revue Economique* 42, 1013-1026. 1-17.

Codognato G, Ghosal S (2000). "Oligopoly à la Cournot-Nash in markets with a continuum of traders," *Discussion Paper No 2000-5*, CEPET (Central European Program in Economic Theory), Institute of Public Economics, Graz University.

D'Arista, Jane (2009). "Financial Concentration" *Wall Street Watch Working Paper No. 3*, August.

Fudenberg, Drew & Tirole, Jean (1986). "A "Signal-Jamming" Theory of Predation," *RAND Journal of Economics*, The RAND Corporation, vol. 17(3), pages 366-376, Autumn.

Gabszewicz J.J. and Vial J.-P. (1972), "Oligopoly 'à la Cournot-Walras' in a general equilibrium analysis," *Journal of Economic Theory* 4, 381-400.

Gaughan, Patrick A. (2002) *Mergers, Acquisitions, and Corporate Restructurings*, 3rd Edition, John Wiley and Sons, New York.

Gomez, Rosario & Goeree, Jacob K. (2008). "Predatory Pricing: Rare Like a Unicorn?," *Handbook of Experimental Economics Results*, Elsevier.

Green EJ, and Porter RH (1984): "Noncooperative Collusion Under Imperfect Price Information," *Econometrica*, 52, 87-100.

Hall, Robert E (1988). "The Relation between Price and Marginal Cost in U.S. Industry," *Journal of Political Economy*, University of Chicago Press, vol. 96(5), pages 921-47, October.

Harrington, Joseph Jr. (1989). "Collusion and predation under (almost) free entry," *International Journal of Industrial Organization*, Elsevier, vol. 7(3), pages 381-401. MacDonald, James (1999) "Concentration & Competition In the

U.S. Food & Agricultural Industries,” Economic Research Service/USDA, Agricultural Outlook, May.

Hendrickson, Mary & Heffernan, William (2007) “Concentration of Agricultural Markets,” <http://usfoodcrisisgroup.org/files/2007-hendrickson-heffernan.pdf>, April, read 10/11/2010.

Lamoreaux, Naomi R. (1991). “Bank Mergers in Late Nineteenth-Century New England: The Contingent Nature of Structural Change,” *The Journal of Economic History*, 51: 537-557.

Levenstein, Margaret (1993). “Price Wars and the Stability of Collusion: A Study of the Pre-World War I Bromine Industry,” NBER Historical Working Papers 0050, National Bureau of Economic Research, Inc.

Lipton, Martin (2006). “Merger Waves in the 19th, 20th and 21st Centuries,” The Davies Lecture, Osgoode Hall Law School, York University, September 14, available at <http://osgoode.yorku.ca/>.

McGee, John (1958). “Predatory Price Cutting: The Standard Oil (N.J.) Case,” *Journal of Law and Economics* 1 (April).

Milgrom, Paul & Roberts, John (1997). “Predation, reputation, and entry deterrence,” Levine’s Working Paper Archive 1460, David K. Levine.

Okuno, Masahiro & Postlewaite, Andrew & Roberts, John (1980). “Oligopoly and Competition in Large Markets,” *American Economic Review*, American Economic Association, vol. 70(1), pages 22-31, March.

Osterdal, Lars Peter (2003). “A note on the stability of collusion in differentiated oligopolies,” *Research in Economics*, Elsevier, vol. 57(1), pages 53-64, March.

Persson, Lars (2004). “Predation and mergers: Is merger law counterproductive?” *European Economic Review*, Volume 48, Issue 2, April, Pages 239-258.

Pot, Erik; Peeters, Ronald; Peters, Hans; Vermeulen, Dries (2010). Intentional Price Wars on the Equilibrium Path, No 028, Research Memoranda, Maastricht : METEOR, Maastricht Research School of Economics of Technology and Organization, <http://halshs.archives-ouvertes.fr/docs/00/37/75/41/PDF/0910.pdf>.

Ray, Debraj (2003). The One-Shot Deviation Principle, mimeo, downloaded Oct 22, 2010. Available, slightly modified, at <http://www.nyu.edu/econ/user/debraj/Courses/05UGGameLSE/Handouts/osdp.pdf> (May 18, 2011).

Roth, David (1996). Rationalizable Predatory Pricing, *Journal of Economic Theory* 68, 380-396.

Shitovitz, B. (1973). “Oligopoly in markets with a continuum of traders,” *Econometrica* 41, 467-501.

Sahi, Siddhartha & Yao, Shuntian (1989). “The non-cooperative equilibria of a trading economy with complete markets and consistent prices,” *Journal of Mathematical Economics*, Elsevier, vol. 18(4), pages 325-346, September.

Yamey (1972). “Predatory Price Cutting: Notes and Comments,” *Journal of Law and Economics*, Vol. 15, No. 1 (Apr), pp. 129-142.

UNCTAD (2008). World Investment Report 2008. United Nations, New York.